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RF CAPTURE IN THE NAL BOOSTER

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The radio frequency accelerating cavities in the booster are turned on abruptly to avoid multipactoring problems. In order that the capture of protons injected from the linac should be adiabatic, the cavities are turned on one at a time about 8 μsec apart. The word "adiabatic" implies only little or no emittance growth and has little to do with the well known adiabatic theorem. The naive application of the adiabatic theorem would lead one to expect an empty bucket with all particles in the unstable region outside. In reality, particles are caught by the growing bucket. Particularly near the unstable fixed points, where the phase motion is slow, this capture can take place whether the rf change is continuous or stepwise. Once particles are inside the bucket the total phase area does depend on the adiabaticity of the process, i.e. on the rate of change of the rf voltage compared to frequency of the phase oscillation. The actual emittance growth must be calculated for a particular rf voltage program.

For a given energy spread the capture efficiency and phase dilution can be expected to depend on at least the final rf

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voltage, the time to full turn-on and the voltage vs time function. Results are given below for a range of each of these three variables. One may also expect space charge effects since most such vary as $\frac{1}{\gamma^2}$ and are therefore greatest at injection. A realistic inclusion of these effects is beyond the reach of the present calculation. Their order of magnitude may be just a few percent in bucket area,[1] however, if perfectly conducting wall model has any meaning at all for the booster case.

The calculations are based on a turn-by-turn integration of the difference equations

$$\phi_{n+1} = \phi_n - \Lambda\varepsilon_n \tag{1}$$

$$\varepsilon_{n+1} = \varepsilon_n + eV_n \sin \phi_{n+1} \tag{2}$$

where

$$\Lambda = 2\ h(1/\gamma^2 - 1/\gamma_t^{\,2})/E_o\gamma\beta \tag{3}$$

and

V_n is the amplitude of the radio frequency,

h is the harmonic number equal to 85 for the booster,

E_o is the proton rest energy in MeV, and

γ_t is the transition gamma equal to 5.445 for the booster.

These equations correspond to a physical situation where there is a single rf gap per turn and thereby overstates the impulsive (nonadiabatic) character of capture in the booster. The more usual differential equations, however, represent the acceleration as uniformly distributed around a turn and may thereby understate the capture losses. By taking steps of less than one turn it would, of course, be possible to treat each booster cavity individually.

The difference equations have been integrated for several sets of parameters. In all cases the injection energy was taken to be 200 MeV with a spread of ±.29 MeV corresponding to $\Delta p/p = \pm 8 \times 10^{-4}$. Common to each sequence of runs is the nominal booster ease where V = .1 MV is reached in sixteen steps taken at 8.4 μsec intervals. Fig. 1 shows the final distribution of particles in the bucket with the original emittance of 3.66 MeV rad shown cross-hatched. The bucket area is

$$A = \frac{eV}{\Lambda} = 5.23 \text{ MeV rad}$$

so that if filled "adiabatically" the final bucket would be 70% filled. Clearly there has been emittance growth during the capture since a sprinkling of points lie quite close to the separatrix. The rms emittance

$$\sigma = \left[\sum_i (\varepsilon_i - \bar{\varepsilon})^2 \sum (\phi_i - \bar{\phi})^2 - \sum (\varepsilon_i - \bar{\varepsilon})(\phi_i - \bar{\phi})\right]^{1/2},$$

where the sum extends over all particles, can be used to define an emittance uniquely for a nonuniform distribution. The rms emittance of the initial bunch is σ = 306 MeV rad. The final distribution in Fig. 1 has σ = .659. Table 1 gives the particle loss and rms emittance for a number of different values of the final rf voltage. If the capture were adiabatic one would expect particle losses to begin at 50 keV/turn rf when the bucket is $1/\sqrt{2}$ as large. Losses start at ~60 keV/turn, however. The emittance values are rather more interesting; the smaller the bucket into which the particles are captured, the less the emittance growth.

A series of cases which differ in the number of steps in the turn-on is tabulated in Table 2. Here one finds that the more smoothly the rf is turned on the greater the rf emittance. This surprising result appears to reflect the filamentation which takes place if the separatrix is moved away from the captured particles slowly so that the effect of the nonlinearity is strong. The last entry in this table is the smoothest that can be obtained with a single cavity.

A series of cases in which the time to full turn-on is varied, taking a step per turn, is tabulated in Table 3. Once again, the less slow turn-on gives the lower rms emittance. Particle losses start where the turn-on time is somewhat less than one longitudinal oscillation period ($\nu_s = .05$). The results of Sessler and Lilliequist(2) based on the differential equation show complete capture down to about one half longitudinal oscillation period for a linear turn-on. This discrepancy, too, shows a distinction between the difference equations and the differential equation. The booster case lies between the two since the cavities are distributed. However, the extrapolation of the sequence in Table 2 suggests that several steps per turn will not lie very close to the continuous case.

The rms emittance figures in the foregoing tables can be considered only as suggestive because even after capture this figure fluctuates considerably. The rms emittance was calculated from a case with sixteen steps to .07 MV at 3 turn intervals after 100, 400 and 1000 turns. The value for σ was .581, .619, and .486, respectively. A measure of phase area less dependent on the scattering of peripheral particles would probably produce a more consistent measure.

To show what is happening in the discrete step capture process four steps in the nominal booster case a phase period apart are shown in Figs. 2-4. The debunched beam is shown in Fig. 2 at the instant the rf comes on. The voltage steps come at 3 turn (8.4 μsec) intervals in increments of .0626 MV. Fig. 3 shows the bunch after 20 turns (7 voltage steps). After 20 turns more the result is as shown in Fig. 4. Many of the particles are seen to be caught in buckets adjacent to the one appearing at their initial position. In the case of a completely debunched beam corresponding particles would be entering the central bucket and the final result is that shown in Fig. 1, which is obtained by taking phases modulo 2π. From a detailed tracing of this kind it is conceivable that one might tailor a few step turn-on which would reduce the emittance growth found in the linear turn-on. One might also consider reducing the capture voltage if sufficient operating stability can be maintained to give reliable capture into the smaller bucket.

REFERENCES

1. S.C. Snowdon, paper to be written.

2. C.G. Lilliequist and K.R. Symon, MURA #491, July 20, 1959.

Table 1

CAPTURE PERCENTAGE AND RMS EMITTANCE AS A FUNCTION OF RF VOLTAGE PER TURN FOR SIXTEEN EQUAL VOLTAGE STEPS AT THREE TURN INTERVALS

Final rf Voltage (MV)	Capture Efficiency (%)	Rms Emittance (MeV rad) at 100 Turns
.100	100.0	.657
.086	100.	.606
.082	100.	.606
.078	100.	.606
.074	99.6	.654
.070	99.6	.581
.066	99.6	.537
.060	99.2	.574+
.050	97.2	.537+

+ at 1000 turns

Table 2

CAPTURE PERCENTAGE AND RMS EMITTANCE AS A FUNCTION OF THE NUMBER OF STEPS IN THE TURN-ON. THE TIME OF THE FINAL TURN-ON IS HELD AT 48 TURNS; THE FINAL VOLTAGE PER TURN IS .100 MV

Number of Steps	Capture Efficiency (%)	Rms Emittance (MeV rad) at 100 Turns
1	82.4	.369
2	88.4	.535
3	88.8	.560
5	100.0	.577
10	100.0	.641
16	100.0	.659
45	100.0	.733

Table 3

CAPTURE PERCENTAGE AND RMS EMITTANCE AS A FUNCTION OF THE RATE OF RF TURN-ON. THE FINAL VOLTAGE IS .1002 MV APPLIED IN EQUAL STEPS AT EACH BEAM TURN

Number Turns to Full On	Capture Efficiency (%)	Rms Emittance (MeV Rad) at 100 Turns
6	86.8	.397
10	91.4	.443
15	96.8	.467
20	99.2	.488
30	100.0	.581
45	100.0	.733

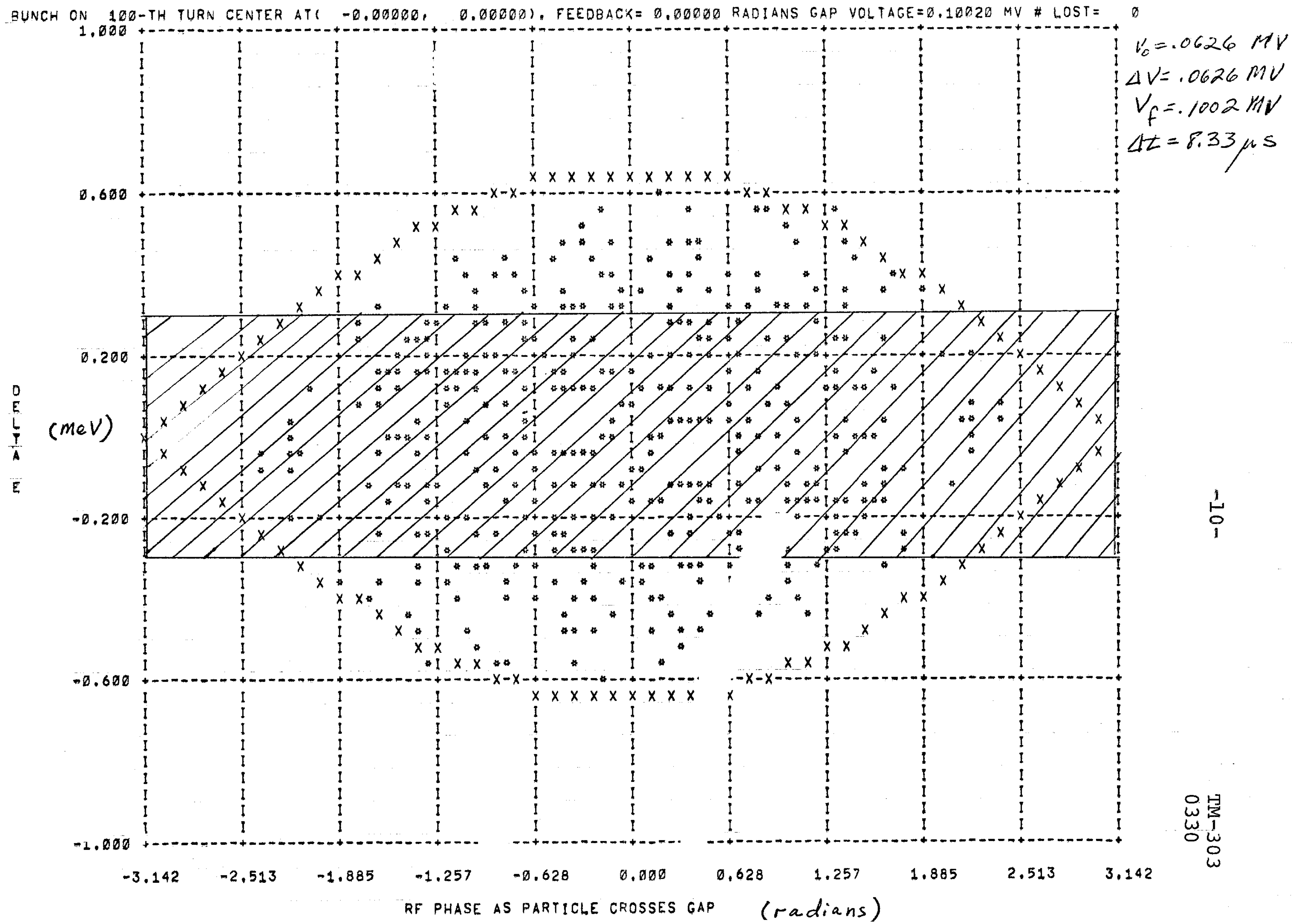

Figure 1. Capture for booster design Case

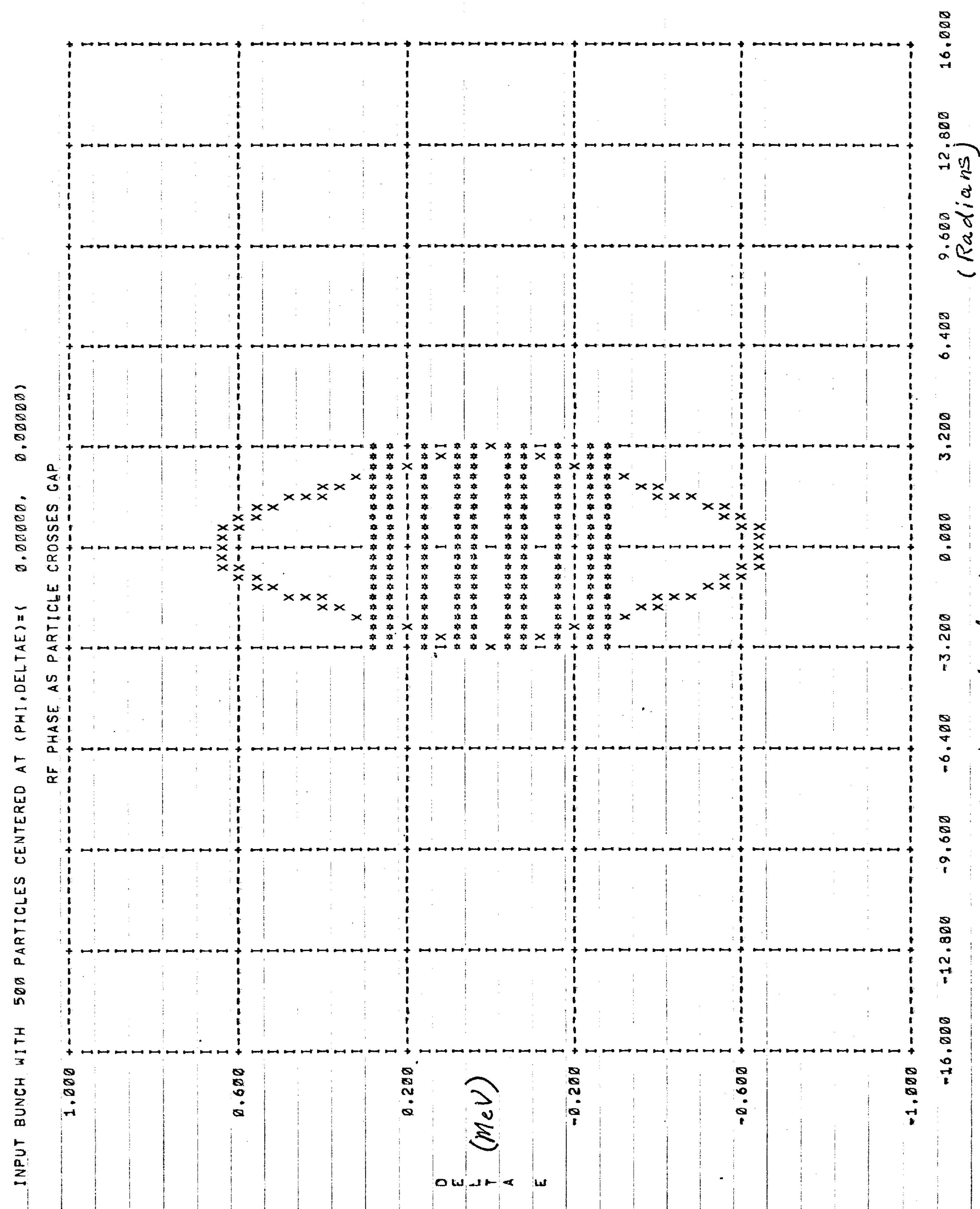

Figure 2: Longitudinal emittance at rf turn on

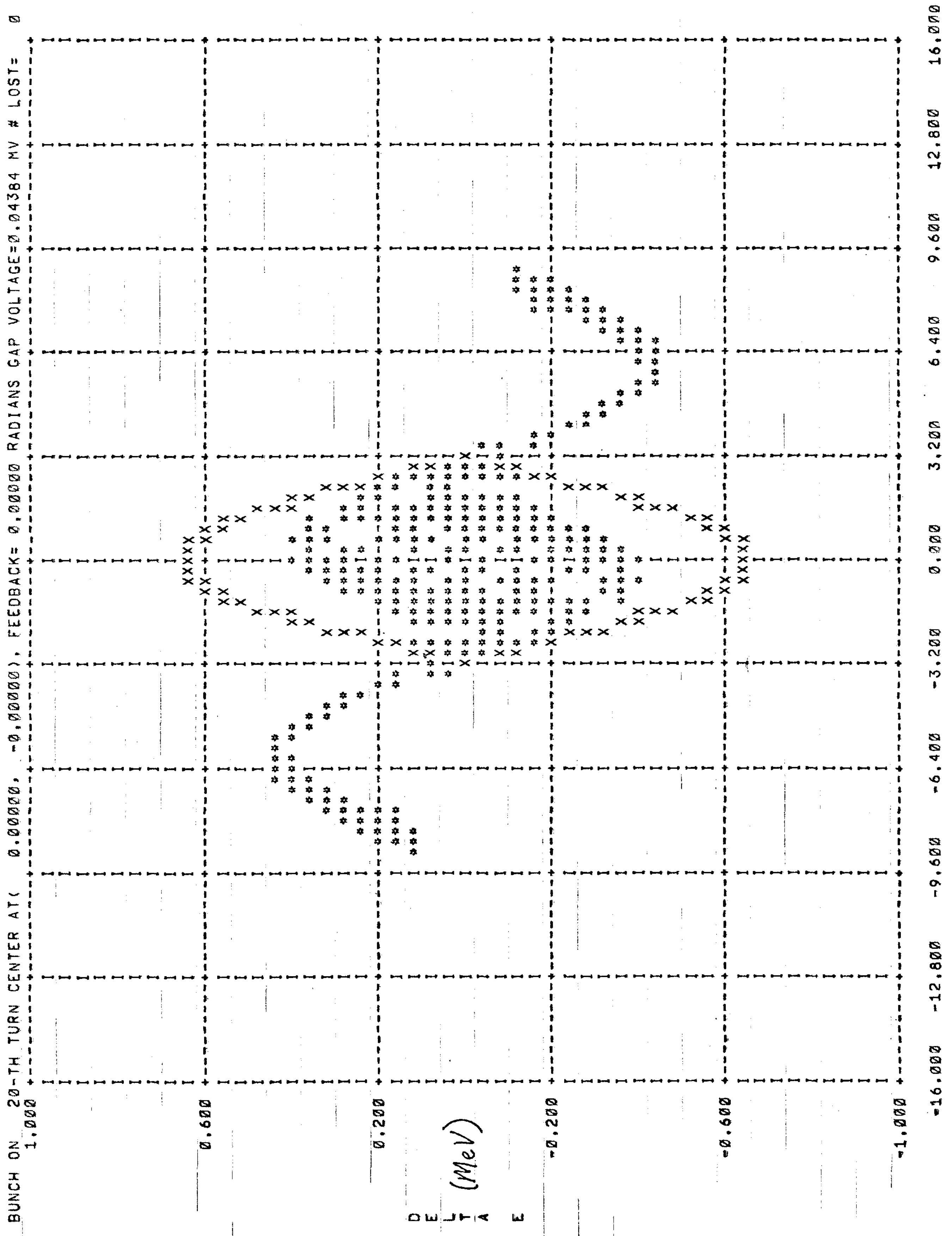

Figure 3: Longitudinal phase distribution after 20 turns with rf on

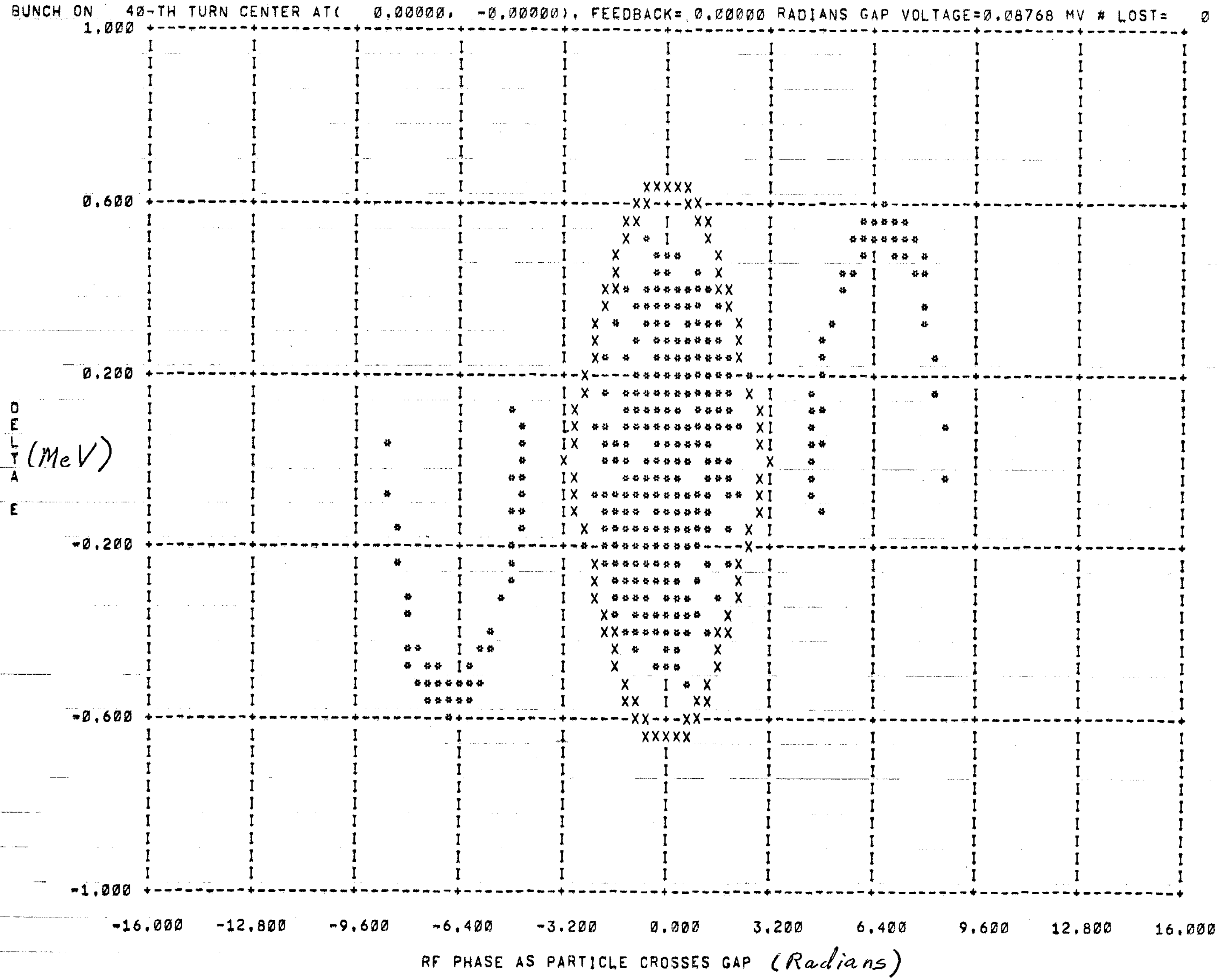

Figure 4: Longitudinal phase distribution after 40 turns with rf on